

On the Architecture of Spacetime Geometry

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We propose entanglement entropy as a probe of the architecture of spacetime in quantum gravity. We argue that the leading contribution to this entropy satisfies an area law for any sufficiently large region in a smooth spacetime, which, in fact, is given by the Bekenstein-Hawking formula. This conjecture is supported by various lines of evidence from perturbative quantum gravity, simplified models of induced gravity and loop quantum gravity, as well as the AdS/CFT correspondence.

Introduction: One of the most remarkable discoveries in fundamental physics was the realization that black hole horizons carry entropy [1]. This entropy is manifest in the spacetime geometry, as expressed by the Bekenstein-Hawking formula:

$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{\mathcal{A}}{4G}, \quad (1)$$

where \mathcal{A} is the area of the horizon. In fact, this result is easily extended to any Killing horizon, including de Sitter [2] and Rindler [3] horizons. Of course, the above result applies for black hole solutions of Einstein's equations (in any number of dimensions). This result can also be extended to account for higher curvature interactions in the gravitational theory and the corresponding expression for the horizon entropy, the 'Wald entropy' [4], again has a geometric character. This celebrated formula (1) draws an unexpected connection between spacetime geometry, thermodynamics, quantum theory and gravity. In fact, interest in this result has long been sparked by the possibility that it provides a window into the nature of quantum gravity. Certainly, reproducing eq. (1) in terms of a microscopic description is now regarded as a necessary hallmark for a consistent theory of quantum gravity.

Of course, this expression (1) also reminds us that the physical constants in Nature can be combined to yield a fundamental length, the Planck scale: $\ell_P^{d-2} = 8\pi G \hbar / c^3$ in d spacetime dimensions. Hence the geometric entropy (1) of the horizon is simply the horizon area measured in units of the Planck scale:

$$S_{geom} = 2\pi \frac{\mathcal{A}}{\ell_P^{d-2}} + \dots \quad (2)$$

Above we have also set Boltzmann's constant to unity and indicated the possibility of contributions subleading to the area term.

In the present paper, we propose that eq. (2) in fact has much wider applicability with the following general conjecture:

In a theory of quantum gravity, for any sufficiently large region in a smooth background spacetime, the entanglement entropy between the degrees of freedom describing the given region with those describing its complement is finite and to leading order, takes the form given in eq. (2).

Of course, an implicit assumption here is that the usual Einstein-Hilbert action (as well as, possibly, a cosmolog-

ical constant term) appears as the leading contribution to the low energy effective gravitational action. The appearance of this geometric entropy (2) in such a general context emphasizes that in quantum gravity, the description of any macroscopic spacetime (even flat Minkowski space) entails a state with a great deal of nontrivial structure in terms of the microscopic degrees of freedom of the theory. Further with this proposal, the area law (2) for entanglement entropy serves as a characteristic signature for the emergence of a semiclassical metric in a theory of quantum gravity. The remainder of the paper will then present various lines of evidence that support our conjecture. We begin with some discussion of the relevant background material.

Entanglement entropy: Entanglement entropy has emerged as a useful measure of entanglement which can be used to characterize the correlations in a quantum system [5]. Given a subsystem A described by the reduced density matrix ρ_A , the entanglement entropy corresponds to the von Neumann entropy of the corresponding density matrix: $S_{EE} = -\text{tr}[\rho_A \log \rho_A]$. In the context of quantum field theory (QFT), when one considers the entanglement between a region and its complement,¹ one finds that the entanglement entropy is UV divergent because of short range correlations in the vicinity of the 'entangling surface' Σ separating the two regions. If the calculation is regulated with a short distance cut-off δ , the leading contributions for a QFT in d spacetime dimensions generically take the form

$$S_{EE} = c_0 \frac{R^{d-2}}{\delta^{d-2}} + c_2 \frac{R^{d-4}}{\delta^{d-4}} + \dots, \quad (3)$$

where R is some (macroscopic) scale characterizing the geometry of Σ . In fact, a closer examination shows that each of these terms has a precise geometric interpretation involving an integration of various factors over the boundary Σ [6]. For example, the leading term yields the famous 'area law' result with $S_{EE} \simeq \tilde{c}_0 \mathcal{A}_\Sigma / \delta^{d-2}$ [7]. Unfortunately the dimensionless coefficients c_k appearing in these power law divergent terms above are sensitive to the details of the UV regulator.

¹ Here and throughout the following, these are spatial regions on a fixed Cauchy surface.

Entanglement entropy has also been discussed in the context of the AdS/CFT correspondence and more broadly, of gauge/gravity duality [8]. Given a particular holographic framework, the entanglement entropy in the $(d-1)$ -dimensional boundary theory between a spatial region A and its complement is calculated by extremizing the following expression

$$S(A) = \frac{2\pi}{\ell_p^{d-2}} \text{ext}_{v \sim A} [\mathcal{A}(v)] \quad (4)$$

over $(d-2)$ -dimensional surfaces v in the bulk spacetime which are homologous to the boundary region A . In particular then, the boundary of v matches the ‘entangling surface’ $\Sigma = \partial A$ in the boundary geometry. Implicitly, eq. (4) assumes that the bulk theory is well approximated by classical Einstein gravity. Hence this expression (4) bears a striking resemblance to black hole entropy, however, in general, the surfaces v do not coincide with an event horizon in the bulk spacetime, or even the boundary of the natural ‘causal wedge’ associated with the boundary region [9]. In the present context, we present this as the first indication that eq. (2) has a wide applicability. Further, we might note that there has already been some speculation that evaluating this expression on other surfaces in the bulk geometry may yield interesting entropic measures of entanglement in the boundary theory [9, 10].

$S_{BH} = S_{EE}$: In fact, the initial proposals [7] to consider what is now called entanglement entropy stemmed from attempts to understand the entropy of black holes. The observation is that (a cross-section of) the horizon plays the role of an entangling surface separating the degrees of freedom between the interior and exterior of the black hole. Hence it was suggested that quantum correlations between these two regions might account for the black hole entropy. Recall that in evaluating the entanglement entropy for a field theory, the leading term obeys the desired area law:

$$S_{EE} = c_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \dots \quad (5)$$

Of course, this is suggestive. In particular, if one had $\delta \simeq \ell_p$ in a quantum theory of gravity, then eq. (5) could reproduce eq. (2). While it is natural to think that gravity should regulate the entanglement entropy, the precise mechanism is unclear [11]. Furthermore, as was commented above, these UV sensitive terms in the entanglement entropy are not universal. That is, different choices of regulator will yield different values for the dimensionless coefficient c_0 .

The latter issue was partially resolved in [12]. There the suggestion was that this area law contribution of ‘low energy’ degrees of freedom actually renormalizes the bare area term: $S_0 = \mathcal{A}/4G_0$ where G_0 is the ‘bare’ Newton’s

constant,² in the sense of perturbative QFT. That is, the renormalization of the Einstein term in the effective action coming from integrating out quantum fields also yields a power law divergence of the form

$$\Delta\left(\frac{1}{G}\right) = \frac{\tilde{c}_0}{\delta^{d-2}}. \quad (6)$$

Here again, the dimensionless coefficient \tilde{c}_0 is also regulator dependent. However, the proposal of Susskind and Uglum [12] is that for a given choice of regulator: $c_0 = \tilde{c}_0/4$. Hence, the area law term in the entanglement entropy across the black hole horizon is precisely $S_{EE} = \frac{\mathcal{A}}{4} \Delta\left(\frac{1}{G}\right)$. While there was a great deal of work done to confirm this idea in explicit examples [13, 14], unfortunately, there were cases where it appeared that the desired matching was not achieved. However, this confusion was recently resolved in [15]. The general arguments there confirm the proposal of [12] for any fields with spin 0, 1/2, 1 and/or 3/2 to all orders in the perturbation expansion of the corresponding relativistic QFT in any static curved spacetime background with a (bifurcate) Killing horizon in any dimension. Unfortunately, there are still technical issues in extending these arguments to spin-2 fields, *i.e.*, the graviton itself. For later purposes, we note that these results confirm that the area term in S_{EE} can be seen as renormalizing the geometric entropy (2) for a Rindler horizon in flat space.

We also refer the interested reader to another recent paper [16], where a related result was presented. The perspective advanced there was that as the renormalization scale is varied, one is simply trading contributions of the horizon entropy between the bare geometric term and the entanglement entropy of the low energy degrees of freedom.

While it seems then that the state of affairs with regards to [12] has been clarified, the situation is still widely regarded as unsatisfactory. The horizon entropy now takes the form:

$$S_{BH} = S_0 + S_{EE} = \frac{\mathcal{A}}{4G_0} + \frac{\mathcal{A}}{4} \Delta\left(\frac{1}{G}\right) + \dots \quad (7)$$

This leaves the question of how to interpret the bare term in the above contribution. One suggestion was to consider models of ‘induced gravity’ [17] in which there is no bare Einstein term, *i.e.*, $1/G_0 = 0$. Rather the entire gravity action is generated by quantum fluctuations of the other degrees of freedom, *e.g.*, some heavy fields. Hence in such a scenario, the entire horizon entropy is also accounted for by the entanglement entropy of the latter degrees of freedom [18]. In fact, we would observe that most modern approaches to quantum gravity take essentially this perspective. While there may be some fundamental description of the theory in terms of microscopic degrees of freedom, graviton excitations, the low

² Here and in the following, we set $\hbar = 1 = c$.

energy Einstein action and the spacetime geometry itself typically arise as some collective or emergent phenomena. With this outlook, it is natural then to view the metric as an effective macroscopic variable [19] and further then the ‘off-shell’ method [20] to calculating horizon entropy is precisely a calculation of the entanglement entropy in terms of these macroscopic variables.³

It should be noted that, in low-energy processes, the change ΔS_{EE} in the entanglement entropy reproduces the Bekenstein-Hawking formula [22],

$$\Delta S_{\text{EE}} = \frac{\Delta A}{4G}, \quad (8)$$

with G being the low-energy Newton constant, and ΔA the change in area of the event horizon. The variation ΔS_{EE} is finite and insensitive to the UV physics because a low-energy process affects only the IR part of the entropy. Moreover it is universal because of the universal coupling of gravitons to the energy-momentum tensor. This result from perturbative quantum gravity provides further support to the idea that the entropy S_{BH} is due to entanglement as in eq. (7).

Of course, it is reassuring to confirm the expectation that $S_{\text{BH}} = S_{\text{EE}}$ with explicit microscopic models. Here one classic example is the eternal AdS black hole, in the AdS/CFT correspondence, where the horizon entropy is interpreted in terms of the entanglement entropy between the microscopic degrees of freedom of the boundary CFT and its thermofield double [23]. In fact, a similar interpretation extends quite generally to asymptotically AdS spacetimes with a Killing horizon, *e.g.*, [21, 24]. Similarly this area law arises from calculations in loop quantum gravity and spin foams, where the entropy of the horizon at the leading order is given by the entanglement entropy of spin-network links crossing the entangling surface [25, 26]. Furthermore, various simplified models of induced gravity also illustrated this point [18, 27, 28].

Entanglement Hamiltonians: Recall that the first step in calculating S_{EE} for some region A was to calculate the density matrix ρ_A . Here we note that an essential role played by ρ_A is to encode the correlators of operators whose support lies in A . In fact, by causality, ρ_A describes such physics throughout the causal development \mathcal{D} of A .⁴ Now since the reduced density matrix is both hermitian and positive semidefinite, it can be expressed as

$$\rho_A = e^{-H_A} \quad (9)$$

for some hermitian operator H_A . In the literature on axiomatic quantum field theory, H_A is known as the ‘modular Hamiltonian’ [29] while the same operator is referred

to as the ‘entanglement Hamiltonian’ in the condensed matter literature [30]. We note that H_A and the unitary operator $U(s) = \rho^{is} = e^{-iH_A s}$ play an important role in axiomatic approaches to establish various formal properties of ρ_A and the algebra of operators on \mathcal{D} . However, we must emphasize that generically H_A is not a local operator and $U(s)$ does not generate a local (geometric) flow on \mathcal{D} . For example, if we begin with a local operator defined at a point, $\mathcal{O} = \phi(x)$, then generally the operator $\mathcal{O}(s) = U(s)\mathcal{O}U^\dagger(s)$ becomes an operator with support over an extended region within \mathcal{D} . To be explicit, we might expect H_A to have the schematic form

$$H_A = \int d^{d-1}x \gamma_1^{\mu\nu}(x) T_{\mu\nu} + \int d^{d-1}x \int d^{d-1}y \gamma_2^{\mu\nu;\rho\sigma}(x, y) T_{\mu\nu} T_{\rho\sigma} + \dots \quad (10)$$

Furthermore, in general, we should expect that other operators beyond the stress tensor can also appear in this expansion. Of course, there are special cases where the modular flow and the modular Hamiltonian are in fact local. A well-known example is given by the Minkowski vacuum state restricted to the Rindler wedge L . That is, taking the entangling surface to be the line $\Sigma = \{t = 0, x = 0\}$ and the region A , the half-space $x > 0$ (and $t = 0$), then the corresponding causal development $\mathcal{D} \equiv L$ is a wedge of Minkowski space. In this case for any QFT, the modular Hamiltonian is just

$$H_A = 2\pi K + c' = -2\pi \int_{x>0} d^{d-2}y dx [x T_{00}] + c' \quad (11)$$

where K is the boost generator in the x direction and c' is a constant introduced to ensure the unit trace of the density matrix. This result is commonly known as the Bisognano-Wichmann theorem [31]. In this special case, the operator $U(s)$ translates operators along the boost orbits within L . Further, of course, interpreted in the sense of Unruh [32], eq. (9) describes a thermal state with respect to this notion of ‘time’ translations.

Now given this formalism, let us turn to the question of calculating the entanglement entropy of some general region. We wish to frame the discussion in the context of a general curved background spacetime in which the curvatures are slowly varying. Within this background, we choose some smooth Cauchy surface and on this surface a smooth entangling surface Σ . We imagine that we are evaluating the entanglement entropy for a collection of quantum fields in this framework. The QFT will be provided with some regulator that introduces a short distance cut-off δ . The latter is much smaller than any geometric scales L_{geom} that arise in defining the background, the Cauchy surface and the entangling surface, *i.e.*, $\delta \ll L_{\text{geom}}$. Now let us zoom in on a small spacetime region Γ of size $L_{\text{reg}} \ll L_{\text{geom}}$ near the entangling surface Σ , as illustrated in figure 1. We imagine that the problem is such that we can choose $\delta \ll L_{\text{reg}} \ll L_{\text{geom}}$. Hence within Γ , the spacetime looks like flat space, the portion

³ Let us note that the ‘off-shell’ method can be extended to higher curvature theories of gravity, in which case it reproduces precisely the Wald entropy. See the discussion around footnote 15 in [21].

⁴ The causal development of A is the set of all points p for which all causal curves through p necessarily intersect A .

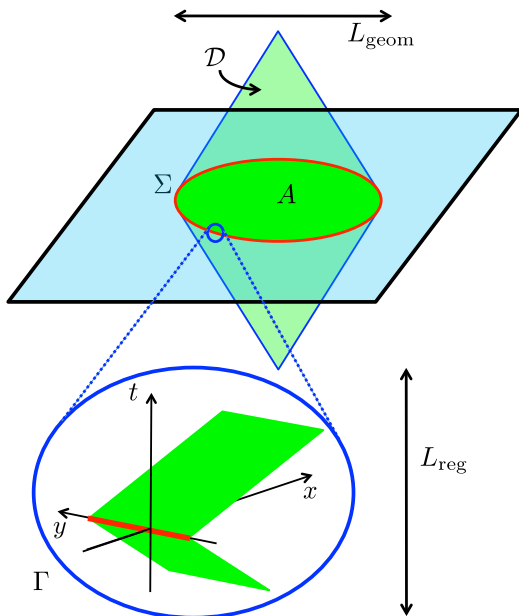


FIG. 1: (Colour Online) Consider a region A on a Cauchy slice of some smooth background geometry. The density matrix ρ_A controls the physics throughout the causal domain \mathcal{D} . This geometry varies only on some large distance scales L_{geom} . We consider a spacetime region Γ of size $L_{\text{reg}} \ll L_{\text{geom}}$ near the entangling surface Σ . Within this region, the spacetime looks like flat space and the light sheets defining $\partial\mathcal{D}$ look like Rindler horizons.

of the entangling surface looks simply like a straight line, and the light sheets defining $\partial\mathcal{D}$ are flat surfaces extending from this line like Rindler horizons.

Now given the general setting in which we have framed our discussion, it would be difficult to consider details of the vacuum state for the QFT without further information. In fact, it may well not be possible to define a unique vacuum state in general. However, rather than focussing on such a precise state, we instead consider general states with the property that correlators in these states reproduce the standard UV singularities of the Minkowski vacuum. For free fields, this is essentially the definition of Hadamard states, and for interacting fields, this Hadamard-like property can be seen as a defining characteristic of the relevant states [33]. For any such states, this ensures that within the region Γ , any correlators have the same form as in Minkowski space up to small corrections. Hence the short distance part of ρ_A that encodes these correlators must have the same structure as in flat space. But then as discussed above, we know precisely the form of this density matrix. In particular, expressing ρ_A as in eq. (9), the leading order contribution to the entanglement Hamiltonian must be precisely the Rindler Hamiltonian, *i.e.*, $H_A = 2\pi K + \dots$

as in eq. (11).⁵

Now this conclusion has two interesting implications. First, we know that the ‘density’ of the entanglement entropy for a Rindler horizon yields a constant leading divergence, *i.e.*, $s \simeq \frac{c_0}{\delta^{d-2}} + \dots$. While the constant c_0 depends on our precise choice of regulator, in the present framework the regulator is fixed and so each element of the entangling surface makes precisely the same contribution. Hence integrating over the entangling surface, we will find the leading singularity will be a contribution of the form $S_{\text{EE}} \simeq c_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \dots$. That is, for any calculation of S_{EE} within the general context set out above, we will find that the leading contribution to the entanglement entropy is an area law term. Furthermore, at this point, we may invoke the results of [15] as applied to Rindler horizons to further relate the constant c_0 to the renormalization of the effective Newton’s constant in this theory. Hence we arrive at the conclusion that the leading contribution to the entanglement entropy takes the form

$$S_{\text{EE}} \simeq \frac{\mathcal{A}_\Sigma}{4} \Delta\left(\frac{1}{G}\right) + \dots \quad (12)$$

That is, for any general region in a smooth background spacetime, as described above, the entanglement entropy of the low energy fields has precisely the necessary form to renormalize the expression given in eq. (2). At the very least, this is a highly nontrivial consistency check of the proposal made at the outset of this paper. That is, in order for eq. (2) to be consistent with the coupling of the perturbative quantum fields to gravity, these low energy degrees of freedom must contribute to the entanglement entropy precisely as in eq. (12) for general regions.

Of course, as with black hole entropy, this discussion leaves open the question of how to interpret the bare area term in S_{EE} , which our proposal requires to be present. Here, we would advocate that applying the ‘off-shell’ method to calculate the Rindler entropy within each region indicates that this bare term must be present as the entanglement entropy of the microscopic gravity degrees of freedom. There will also be many higher order corrections, *e.g.*, corresponding to integrals of both intrinsic and extrinsic curvatures over the entangling surface. It is not clear that all of these will be associated to the renormalization of various gravitational couplings [34, 35].

Of course, as before, it is much more satisfying to explicitly realize the desired result with calculations within a given microscopic model. Here one can draw

⁵ As an aside, let us note that this result agrees with the usual intuition about the local character of UV physics. In particular, as stressed above, the full entanglement Hamiltonian will generally be a nonlocal operator but here we have argued that the leading UV sensitive contribution to S_{EE} is controlled by a local term in H_A . The same intuition suggests that it is natural to think that the other UV divergent contributions to S_{EE} should also be governed by local terms in H_A [34].

evidence from two sources: First, the Randall-Sundrum II braneworld [36] provides an example of an induced gravity model. In particular, it has been observed that in this framework, using holographic prescription for entanglement entropy [8], black hole entropy corresponds to entanglement entropy [27]. However, it is also straightforward to show that the area term in eq. (2) appears for any sufficiently large region, irrespective of whether or not the entangling surface corresponds to an event horizon [27, 35]. Similar results were noted in [28] for other simple induced gravity models using heat kernel techniques. Finally we turn to loop quantum gravity to find support for our conjecture.

Spin Foam Models: In ‘loop quantum gravity’, a smooth macroscopic geometry is expected to emerge from a description of space and spacetime which is discrete at a fundamental level [37]. There has been recent progress in the understanding of black hole entropy in this context [25, 38] and so it is natural to ask whether these models give some evidence for our conjecture that general regions of macroscopic spacetimes carry an entanglement entropy given by eq. (2).

Consider a cellular decomposition of a three-dimensional manifold, for instance, a triangulation. A spin-network graph with a node in each cell and a link connecting nodes in neighbouring cells is said to be dual to this triangulation. Lorentz-group representations label the links of the graph and determine a quantum geometry of the triangulation. Generically such states are highly entangled [26]. In particular, we consider the vacuum state defined using the covariant spinfoam dynamics, which has the properties that it is invariant under local Lorentz transformation and time translations. Now, even neglecting interactions between different links, the state has entanglement associated to the endpoints of each link. In the cellular picture, the quantum geometries of two nearby cells in the three-dimensional manifold are entangled.

Now we consider a three-dimensional region A in the manifold. The cellular decomposition induces on the boundary Σ of the region a tessellation in two-dimensional cells. In the dual picture these are links l crossing the surface Σ . Exactly as discussed above, the relevant part of the reduced density matrix ρ_A can be written in the form (9) with the entanglement Hamiltonian

$$H_A = 2\pi \sum_l K_l + \log Z. \quad (13)$$

The sum is over the links l that cross the entangling surface Σ , and K_l is the hermitian generator of boosts in the unitary representation of the Lorentz group associated to the link. This expression has the same form of eq. (11) for the QFT case. The term $\log Z$ provides the normalization of the density matrix $\rho_A = e^{-H_A}$, *i.e.*, this term provides the constant c' in eq. (11). The entanglement entropy is now easily computed:

$$S_{EE} = -\text{Tr}(\rho_A \log \rho_A) = 2\pi \text{Tr}(\sum_l K_l \rho_A) + \log Z. \quad (14)$$

The simplicity constraint on representations of the Lorentz group allows us to express the first term as the area \mathcal{A}_Σ of the entangling surface [25]. The second term is proportional to the number \mathcal{N} of links crossing Σ , so that we have

$$S_{EE} = \frac{\mathcal{A}_\Sigma}{4G_0} + \mu(\gamma) \mathcal{N}, \quad (15)$$

where μ is a chemical potential that depends on the Immirzi parameter γ [38]. The entanglement entropy is finite because the theory has no degrees of freedom below the scale $\ell_{LQG} = (8\pi\gamma G_0)^{1/2}$, the physical cut-off scale in loop quantum gravity. As the area \mathcal{A}_Σ is proportional to \mathcal{N} , the second term can be understood as a finite renormalization of G_0 and be reabsorbed in the first term in the same way as described in eq. (7), thus providing further evidence for our conjecture.

Discussion: We have proposed that the Bekenstein-Hawking formula has a much wider applicability that previously considered. In fact, our conjecture is that eq. (2) corresponds to the leading contribution to the entanglement entropy for any sufficiently large region in a theory of quantum gravity. Evidence for this conjecture was presented from four directions:

- i) In the AdS/CFT correspondence, the well-tested prescription for holographic entanglement entropy [8] clearly assigns an entropy to large classes of surfaces which are unrelated to horizons, with precisely eq. (2) as the leading term.
- ii) In examining quantum fields in curved spacetime, for any large region, the leading contribution to the entanglement entropy is an area term and the coefficient of this term matches precisely the renormalization of Newton’s constant in eq. (2). Further, applying the ‘off-shell’ method to calculate the Rindler entropy locally along the entangling surface suggests the presence of a bare term $\mathcal{A}/4G_0$, as well.
- iii) In simplified models of induced gravity, the leading term to the entanglement entropy for large regions is finite and takes precisely the form given by eq. (2) [27, 28, 35].
- iv) Our preliminary investigations of spin foam models indicate that general regions will carry a finite entanglement entropy, again with the leading term described by eq. (2).

We feel that combining these results provides strong evidence for our conjecture as a general result.

Our proposal demands that quantum gravity effects two essential features for entanglement entropy: First, it ‘regulates’ entanglement entropies for general regions. This might be seen as another realization of the general lore that quantum gravity contains fewer states than quantum field theory. The second property is that this regulator yields a simple universal result, *i.e.*, eq. (2). This property would seem to rely on the universal couplings of the effective Einstein theory emerging at low energies [22]. We expect this universality is a unique feature of the entanglement entropy. For example, the Rényi

entropies [39, 40], which would provide another measure of the entanglement between regions, should also exhibit an area law behaviour at leading order. However, the precise coefficient of the area term would likely depend on the microscopic details of the underlying quantum gravity theory.

In quantum many-body systems, the entanglement entropy typically satisfies an area law [41]. However, this is not the typical behaviour for generic states in the full Hilbert space [42]. Hence it seems that the locality of the underlying Hamiltonian restricts the entanglement of the microscopic constituents in the low energy states of these systems. This feature was central to the recent development of tensor network techniques to better understand the nature of quantum matter [43]. Drawing an analogy here, we expect that generic states in the full Hilbert space of quantum gravity will not correspond to anything resembling a smooth spacetime. Rather states describing smooth macroscopic spacetimes require a certain structure for the short-range entanglement such that we get the area law behaviour (2) as conjectured here. Hence this discussion suggests that the area law entanglement of eq. (2) can be regarded as a signature of states that approximate smooth spacetimes in quantum gravity. In this way then, eq. (2) provides us with a glimpse into the quantum architecture of macroscopic spacetime geometries.

In closing, we consider some future directions: Clearly, it is of interest to further develop the calculations for the spin foam models. Given the discussion in the preceding

paragraph, it would be interesting if the tensor network approaches in condensed matter could provide new lessons on how to deal with discrete models of quantum gravity. It would also be interesting to identify analogous calculations in the context of string theory, as well as to better understand our proposal in a holographic framework. Moreover, it would be useful to identify if this entanglement entropy has an operational meaning. Certainly, when applied to black hole horizons, there is an interpretation in terms of thermodynamic entropy of the black hole, where it should also correspond to a counting of states. Here we might note that low-energy perturbations of the entanglement entropy seem to admit such a thermodynamic interpretation [22, 44].

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